Analysis of a Queueing System under Deadline and Workload Controls

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Abstract—In this paper, we analyze a special queueing system that has workload control and a deadline of the data. The idle server starts the busy period whether the total amount of accumulated workload during the idle period exceeds the predetermined threshold or the remaining deadline of a data becomes to zero. After busy period, the server takes the deactivation delay. If a customer arrives during the deactivation delay, server starts its service immediately. From the viewpoint of energy optimization, we derive the mean length of an arbitrary cycle and then also derive the mean energy consumption of a cycle. With these results, the optimal control parameter which minimizes the energy consumption per unit time can be obtained.

Keywords—queueing system; deadline; server control policy; energy optimization

I. INTRODUCTION

Battery power of mobile devices is one of the most crucial resources for mobile devices. Therefore, power consumption is a big concern in mobile environments. Unfortunately, the performance of battery power fails to meet the power needs of high-end mobile devices [1]. Efficient power management of wireless interfaces should be employed for mobile devices. Dynamic power management is widely employed in order to support multiple power modes such as active modes (e.g., transmit, receive, and idle mode) and inactive modes (e.g., sleep mode and power off) [2]. Therefore maximizing the staying time of the wireless interface in an inactive mode is an essential scheme to reduce energy expenditure. However, required overhead energy and time for turning on/off the wireless interface are not negligible.

In this study, we analyze a special queueing system that has workload control and a deadline of the data. The idle server starts the busy period whether the total amount of accumulated workload during the idle period exceeds the predetermined threshold or the remaining deadline of a data becomes to zero. After busy period, the server takes the deactivation delay. If a customer arrives during the deactivation delay, server starts its service immediately. From the viewpoint of energy optimization, we derive the mean length of an arbitrary cycle and then also derive the mean energy consumption for a cycle. With these results, we can find the optimal control parameter which minimizes the energy consumption per unit time.

This paper is a theoretical study to support the energy efficient carpool policy which is proposed in [3].

II. ANALYSIS OF QUEUEING SYSTEM

In this paper, we assume that the arrival process is Poisson process with rate λ, service time distribution is exponential distribution with parameter μ, and the deadline of each customer is exponentially distributed with parameter η. Although the analysis of this system is based on an M/M/1 queue, but there are some complexities in calculation. These figures are referred to [3].

For the analysis, let us denote some symbols and probabilities as follows:

$C_{A(B)}$: time length of the cycle correspond to Case A(B)
$I_{A(B)}$: time length of an idle period in Case A(B)
$B_{A(B)}$: time length of a busy period in Case A(B)
$AD$: time length of an activation delay, a fixed value
$DD$: time length of a deactivation delay, a fixed value
$IT$: time length of an idle timeout period, a random variable
$T$: time length of an idle timeout, a fixed variable
$N_{A(B)}$: number of data at the start of busy period in Case A(B)
$U_{A(B)}$: total workload at the start of busy period in Case A(B)
$S$: service time of an arbitrary customer
$S_n$: n-th convolution of service time $S$
$F$: distribution function of $S$
$F_n$: distribution function of $S_n$
$E(\cdot)$: expectation of a random variable
$M(t)$: renewal function

The probability that an arbitrary cycle is case A or case B is

\[ P(\text{Case } A) = \exp(-\eta D) / (\lambda + \eta), \]
\[ P(\text{Case } B) = 1 - \exp(-\eta D) / (\lambda + \eta). \]  

(1)

Now we can obtain the mean cycle length of ‘Case A’ with

\[ E(C_A) = E(I_A) + AD + E(B_A) + E(IT) + DD. \]  

(2)

From Fig.1, we can observe that the operational behavior of case A cycle is similar to a queueing system under D-policy [4]. Also, in case of ‘A’ cycle, the inter-arrival time of a data which arrives during the idle period of case A is shorter than the deadline of all waiting data which arrive earlier than this newly arriving data.
The idle period of \( E \) is a random variable because it has a length of \( T \), or the sum of inter-arrival time which is less than \( T \). busy period of ordinary M/M/1 queue, and expected value of \( IT \). Considering all possible cases and by the total expectation, we have

\[
E(IT) = (1-\exp(-\lambda T)) / (1 - \lambda E(S))(\lambda \exp(-\lambda T)).
\]  

From (3)-(5), we complete (2).

Similarly, the mean cycle length of ‘Case B’ can be obtained by

\[
E(C_B) = E(I_a) + AD + E(B_a) + E(IT) + DD.
\]  

The idle period of ‘Case B’ cycle is

\[
E(I_a) = 1/\lambda + \sum_{n=0}^{\infty} F_n(D) / (n\mu + \lambda).
\]  

Mean length of busy period is a delay cycle which has a delay with total workload at the busy period, so we have

\[
E(B_a) = E(S)M(D) / (1 - \lambda E(S)).
\]  

Both type of cycle (Case A and Case B) has the same situation during idle timeout period, so we can use (5) to complete (6). From (1), (2) and (6), we finally have the mean length of an arbitrary cycle as follows:

\[
E(C) = 1/\lambda + \sum_{n=0}^{\infty} F_n(D) / (n\mu + \lambda) + AD
+ E(S) [M(D) + \exp(-\eta D/\eta + \mu)] / (1 - \lambda E(S))
+ (1-\exp(-\lambda T)) / (1 - \lambda E(S))(\lambda \exp(-\lambda T)) + DD
\]  

III. ANALYSIS OF ENERGY CONSUMPTION

In this section, we analyze the mean energy consumption of an arbitrary cycle. We assume that there are some relationships among the energy consumption for each time period as follows:

\[
E_{AD} > E_{DD} = E_{TX} = E_T > E_N.
\]

where \( E_{AD}, E_{DD}, E_{TX}, \) and \( E_T \) is the average energy consumption during activation delay (AD), deactivation delay (DD), busy period, and idle timeout (T), respectively. \( E_N \) is the average energy consumption during an idle period and this value is relatively very smaller than others.

Let us denote \( \Gamma_{A(B)} \) as the conditional expectation of energy consumption of ‘Case A(B)’ cycles. Then, we have

\[
\Gamma_A = E(Ia) E_N + AD \cdot E_{AD} + E(B_a) \cdot E_{TX} + E(\text{IT}) + DD \cdot E_{DD}.
\]  

We already derived all mean lengths of each period in section II, so now we need to find expectation of energy consumption during idle timeout period (IT) and it is

\[
E_N = \exp(\lambda T) + (1 - \lambda E(S)) E_r + E(S) E_{TX} - 1
\]  

Note that \( T \) is a fixed value but \( IT \) is a random variable.

In the same way, we can obtain \( \Gamma_B \) and the average energy consumption of an ordinary cycle becomes

\[
\Gamma = \Gamma_A \times P(\text{Case A}) + \Gamma_B \times P(\text{Case B}).
\]  

From (12) and (9), we finally have the expected energy consumption per unit time to solve the energy optimization problem.

IV. CONCLUSIONS AND FURTHER STUDY

In this paper, we studied a flexible queueing system which starts busy period whether the deadline of arriving data during the idle period ends or the accumulated workload of data exceeds predetermined threshold \( D \), whichever occurs first. We first analyze the system in the viewpoint of queueing approach and then analyze the energy expenditure during a cycle.

We assumed the queueing model as an M/M/1 queue but this would be expanded to the case of general service time. For the further study, analysis of mean performance measures from the system equations could be considered.

REFERENCES


