Direction Finding Based on Cuckoo Search Algorithm in the Strong Impulse Noise

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Abstract—In order to resolve the difficulty of direction finding in the strong impulse noise, based on maximum likelihood (ML) function and infinite norm, a infinite-norm maximum likelihood (IML) approach is proposed. The proposed approach can reduce the impact of impulse noise and improve the performance of the original ML algorithm significantly. In order to obtain the global optimal solution of the proposed IML approach, cuckoo search (CS) algorithm is applied to solve the objective function of IML, which can enhance the capability of searching and improve the relationship between exploration and exploitation, and a novel direction finding approach called CS-IML is proposed. The proposed CS-IML offers a promising alternative to the conventional approaches in impulse noise, and its merits lie in the fact that it avoids stagnation and has the robust performance. Monte-Carlo simulations have shown that the proposed CS-IML has high success rate of estimation and the capability to find coherent and independent signal sources in the strong impulse noise.

Keywords—direction finding; impulse noise; infinite norm; maximum likelihood algorithm; cuckoo search

I. INTRODUCTION

Direction finding is a major problem of interest in array signal processing, which has a wide range of applications in the areas of radar, sonar, seismology, wireless communication and so on. In the past decades, many useful direction finding approaches have been proposed, which mainly includes the multiple signal classification (MUSIC) [1], the estimation of signal parameters via rotational invariance technique (ESPRIT) [2] and the maximum likelihood (ML) [3] algorithm. In general, these direction finding approaches are designed in the presence of Gaussian noise, and the development of the Gaussian model has been mature. However, in some scenarios, many random signals or noises do not obey the Gaussian distribution, such as the lightning atmospheric noises, the line instantaneous peak voice signals of the communications, the underwater acoustic signals as well as a variety of man-made noises. There are significant peak values in these signals or noises, which can be described by the symmetric α-stable (SaS) process with different characteristic exponents [4].

Because the second and higher order moments of SaS distribution do not exist, a robust covariation (ROC) matrix and the ROC-MUSIC approach has been proposed in [5]. Moreover, the fractional low order moment (FLOM) was presented in [6] and FLOM-MUSIC approach was proposed in the presence of impulse noise. Although the above two basic approaches are widely employed in the presence of impulse noise, their estimation performance is not really good, especially in the strong impulse noise. In order to improve the robustness of direction finding approaches in the strong impulse noise, based on ML algorithm and infinite-norm normalization, we propose the infinite-norm maximum likelihood (IML) approach.

However, the IML approach involves a multidimensional nonlinear optimization problem with an increased computational burden. One of appropriate approaches is to employ meta-heuristics to solve the IML function, which is capable of reducing the computational complexity. Because it could be regarded as a continuous optimization problem, the genetic algorithm (GA) [7] and the particle swarm optimization (PSO) [8] have been used to find directions of signal sources. However, the performance of these meta-heuristics is not good in some scenarios. They have the weakness of precocity and may plunge into the local optimal solution. Recently, cuckoo search (CS) algorithm inspired by the brood parasitism strategy in some cuckoo species was proposed by Yang and Deb [9]. The most powerful feature of CS is the application of Lévy flights [10], by which CS can enhance the capability of searching and improve the relationship between exploration and exploitation.

In this paper, in order to obtain the global optimal solution of the proposed IML approach, CS is employed to solve the objective function of IML approach, then a novel direction finding approach called CS-IML is proposed in the presence of impulse noise. Simulation results have shown that the proposed CS-IML has the capability to obtain high accuracy and success rate of estimation for independent and coherent signal sources in the strong impulse noise, which offers a promising alternative to the conventional approaches of direction finding.

II. INFINITE-NORM MAXIMUM LIKELIHOOD APPROACH FOR DIRECTION FINDING IN THE PRESENCE OF IMPULSE NOISE

Consider $M$ narrowband far-field signal sources with wavelength $\lambda$ impinging on a uniform linear array (ULA), which contains $N$ ($N>M$) isotropic sensors equally spaced with distance $d$. The number of signal sources is assumed to be estimated by other approaches and the directions of the $M$ signal sources are denoted by $\theta = [\theta_1, \theta_2, ..., \theta_M]$. The classical mathematical model of direction finding can be written as

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where \( y(k) \) represents the \( N \times 1 \) signal vector received by the array for the \( k \)th snapshot, \( A(0) = [a(\theta_1), a(\theta_2), ..., a(\theta_M)] \) is the \( N \times M \) array manifold with the steering vector \( a(\theta_m) = [1, e^{j2\pi \sin \theta_1}], ..., e^{j2\pi (N-1) \sin \theta_M}] \) \((m = 1, 2, ..., M)\), \( x(k) \) represents the \( M \times 1 \) transmitted signal vector for the \( k \)th snapshot, \( n(k) \) is the \( N \times 1 \) complex impulse noise vector for the \( k \)th snapshot which obeys the standardized SNR distribution with characteristic exponent \( \alpha \), and \( K \) is the number of snapshots.

Because the second and higher order moments of SNR distribution do not exist, the infinity-norm normalization of received data can be denoted as \( \hat{y}(k) = y(k) / \max_{1 \leq i \leq N} |y_i(k)| \). In the case of a finite number of snapshots, the covariance matrix of weighted signal \( \hat{y}(k) \) can be expressed as

\[
\hat{R}_y = \frac{1}{K} \sum_{k=1}^{K} \hat{y}(k)\hat{y}^H(k),
\]

where superscript \((\cdot)^H\) denotes conjugate transpose.

Thus, the IML estimation value of \( \theta \) can be obtained by solving

\[
\hat{\theta} = \arg \max_{\theta} \text{tr}[P_{A(0)} \hat{R}_y],
\]

where \( \text{tr}[\cdot] \) represents the trace of a matrix, \( P_{A(0)} = A(0)[A^H(0)A(0)]^{-1}A^H(0) \) is the projection matrix of the array manifold \( A(0) \).

III. INFINITE-NORM MAXIMUM LIKELIHOOD APPROACH BASED ON CUCKOO SEARCH ALGORITHM FOR DIRECTION FINDING

CS algorithm is inspired by the brood parasitism strategy in some cuckoo species. In order to search for suitable nests to lay eggs, some cuckoos lay their eggs in other host birds’ nests and may remove eggs of host birds to survive. However, host birds may discover the eggs of cuckoos with a probability \( p_s \in [0, 1] \), then throw them out or build a new nest at another place. In the implementation process of CS, a population \( E' = [e'_1, e'_2, ..., e'_Q] \) has \( Q \) eggs and evolves from the initial generation \((i=0)\) to the maximum number of iterations. The \( i \)th egg \( e'_i = [e'_{i,1}, e'_{i,2}, ..., e'_{i,P}] \) \((i = 1, 2, ..., Q) \) represents a \( P \)-dimensional solution and is stored in a nest. Each dimension of an egg corresponds to the IML estimation value and \( P = M \). In order to evaluate the quality of the egg \( e'_i \), the fitness function can be defined as \( f(e'_i) = \text{tr}[P_{A(e'_i)} \hat{R}_y] \), whose value denotes the fitness of \( e'_i \). The algorithm is composed of three operators, i.e., (A) Lévy flight operator, (B) new nests built operator and (C) greedy selection operator.

A. Lévy Flight Operator

The most powerful feature of CS is the application of Lévy flights rather than simple isotropic random walks. In order to generate a new egg \( g'_i \) \((i = 1, 2, ..., Q)\), it may perturb the current egg \( e'_i \) with a position change \( c_i \), which can be obtained by a random step \( s_i \). The random step \( s_i \) obeys a symmetric Lévy distribution, and Mantegna’s algorithm [11] is used to produce \( s_i \) according to (4).

\[
s_i = \frac{w}{|v|^{1/\alpha}},
\]

where \( w = [w_1, w_2, ..., w_P] \) and \( v = [v_1, v_2, ..., v_P] \) are \( P \)-dimensional vectors, \( \alpha = 3/2 \).

Each element of \( w \) and \( v \) obeys the normal distribution according to (5).

\[
w_j \sim N(0, \sigma_w^2), v_j \sim N(0, \sigma_v^2), j = 1, 2, ..., P,
\]

where \( \sigma_w = \left[ \frac{\Gamma(1+\epsilon) \cdot \sin(\pi \epsilon/2)}{\Gamma(1+\epsilon/2)} \cdot 2^{(\epsilon-1)/2} \right]^{1/\epsilon} \)

and \( \sigma_v = 1, \Gamma(\cdot) \) denotes the gamma distribution.

Then, the position change \( c_i \) is calculated in the following.

\[
c_i = 0.01 \cdot s_i \oplus (e'_i - e^{best}),
\]

where \( \oplus \) represents entry-wise multiplications, \( e^{best} \) denotes the global optimal solution seen so far according to fitness.

Finally, the new egg \( g'_i \) can be updated as

\[
g'_i = e'_i + c_i.
\]

B. New Nests Built Operator

In this operator, the egg \( g'_i \) \((i = 1, 2, ..., Q)\) may be discovered with the probability \( p_s \in [0, 1] \), and a new nest is built at another place to replace the old one. Thus, the operator can be described in the following:

\[
g'_i = \begin{cases} g'_i + \text{rand} \cdot (g'_d - g'_s), & \text{if } r < p_s, \\ g'_s, & \text{else}, \end{cases}
\]

where \( \text{rand} \) is a uniform random number in the range of \([0, 1]\), \( d_1 \) and \( d_2 \) are random integers from 1 to \( Q \), \( r \) is also a uniform random number in the range of \([0, 1]\).

C. Greedy Selection Operator

After executing operator A or operator B, the greedy selection operator should be employed, which can accelerate the maturity of cuckoos. In this operator, if the fitness value of
the new egg is better than the old one, the new solution will be accepted and retained. Thus, the operator can be described as follows:

\[ g'_i = \begin{cases} g'_i, & \text{if } f(g'_i) > f(e'_i), \\ e'_i, & \text{else}. \end{cases} \]  

(9)

\[ e'^{i+1}_j = \begin{cases} e'^{i+1}_j, & \text{if } f(e'^{i+1}_j) > f(g'_i), \\ g'_i, & \text{else}. \end{cases} \]  

(10)

D. Complete CS-IML Approach for Direction Finding
As we described above, each individual (egg) represents a candidate solution of IML approach. In the process of implementing, operators A, B and C are repeatedly used until the number of iterations has been reached. Thus, the complete CS-IML approach for direction finding can be described as follows:

Step 1: Initialization. Firstly set the parameters of CS: the population size \( Q \), the discovery probability \( p_a \) and the number of iterations. Then randomly initialize the \( i \)th individual \( e^0_i = [e^0_{i1}, e^0_{i2}, \ldots, e^0_{iP}] \) with \( P \)-dimensional solution space, and the \( j \)th dimension of the \( i \)th individual is initialized as

\[ e^0_{i,j} = b^\text{low}_j + \text{rand} \cdot (b^\text{high}_j - b^\text{low}_j), \quad j = 1, 2, \ldots, P, \]  

(11)

where \( b^\text{low}_j \) represents the initial lower bound, \( b^\text{high}_j \) represents the initial upper bound, and \text{rand} is a uniform random number in the range of \([0, 1]\).

Step 2: Calculate the fitness values of all the individuals of the initial population, and select the individual with optimal fitness value as the current global optimal individual.

Step 3: Generate new individuals within the range of upper and lower boundaries by executing operator A, and calculate corresponding fitness values of new individuals. Then accept the better individuals by executing operator C and find the current optimal individual according to their fitness values.

Step 4: Replace some individuals within the range of upper and lower boundaries by executing operator B, and calculate corresponding fitness values of new individuals. Then accept the better individuals by executing operator C and find the current optimal individual according to their fitness values.

Step 5: If the current optimal individual is better than the global optimal individual, the global optimal individual will be replaced for the next generation.

Step 6: Check the termination condition: if satisfied, terminate the loop and output the global optimal solution, which represents the estimated value of CS-IML approach for direction finding; otherwise, return to Step 3.

IV. SIMULATION RESULTS AND EVALUATIONS
In the section, we will present a series of simulations to assess the relevant performance of the proposed CS-IML in the presence of impulse noise. Consider that direction finding system is equipped with 10 antennas, which constitute the ULA with half-wavelength inter-element spacing. The number of snapshots \( K \) is set to 500. The impulse noise is assumed to obey the standard SnS distribution with characteristic exponent \( a \), and the generalized signal-to-noise ratio (GSNR) is computed by \( \text{GSNR} = 10 \log_{10} \left( \frac{E(|x(k)|^2)}{\gamma} \right) \), where \( E(|x(k)|^2) \) represents the average power of signals, \( \gamma \) represents the dispersion parameter (\( \gamma > 0 \)). For CS, some parameters are set as follows: the size of population \( Q=25 \), the discovery probability \( p_a = 0.25 \) and the number of iterations is set to 100.

In the following, we compare CS-IML with some previous classical direction finding approaches, such as ROC-MUSIC [5] and FLOM-MUSIC [6]. Consider two statistically independent signal sources impinging on the array from the directions of \([\theta_1=49^\circ, \theta_2=61^\circ]\), and the number of Monte-Carlo simulations \( N_{MC} \) is 500. Fig. 1 shows the root mean square error (RMSE) of three kinds of approaches when GSNR varies from 0 to 25dB with \( \alpha=1.5 \). The RMSE can be defined as \( \text{RMSE} = 1/N_{MC} \cdot \sum_{j=1}^{N_{MC}} \sqrt{\frac{1}{M} \cdot \sum_{n=1}^{M} (\theta_i - \hat{\theta}_j)^2} \), where \( \theta_i \) represents the real value of the \( i \)th direction, \( \hat{\theta}_j \) represents the estimated value of the \( i \)th direction in the \( j \)th simulation. Fig. 2 shows the success rate curves of three kinds of approaches when \( \alpha \) varies from 0.6 to 2 with GSNR=20dB. In the paper, when absolute value of estimation error is less than one degree, we regard it as a successful estimation. And success rate is computed by the proportion of successful estimations. Moreover, Fig. 1 shows that the CS-IML can obtain more accurate estimation performance compared with ROC-MUSIC and FLOM-MUSIC for independent signal sources in the presence of impulse noise. Fig. 2 shows that the CS-IML is superior to ROC-MUSIC and FLOM-MUSIC in terms of success rate for independent signal sources, especially in the strong impulse noise, which is because that it takes advantage of the infinity-norm normalization to resist the impulse noise.

![Fig. 1. Curves of RMSE versus GSNR for independent signal sources with \( \alpha=1.5 \).](image-url)
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ESPRIT.

Fig. 3. Estimation results for coherent signal sources with \( \alpha=1.5 \).

Fig. 4. Estimation results for coherent signal sources with \( \alpha=0.9 \).

Fig. 2. Comparison of success rate versus characteristic exponent for independent signal sources with GSNR=20dB.

Then the simulations were carried out for three coherent signal sources from the directions of \( \{ \theta_1 = 36^\circ, \theta_2 = 42^\circ, \theta_3 = 51^\circ \} \), and the number of Monte-Carlo simulations is 50. Fig. 3 and Fig. 4 show the estimation results of CS-IML for three coherent signal sources when \( \alpha \) is set to 1.5 and 0.9, respectively. We can observe from Fig. 3 and Fig. 4 that the directions of the coherent signal sources can be correctly estimated. Furthermore, the robustness and superiority of CS-IML are verified in the strong impulse noise.

V. CONCLUSIONS

In the paper, in order to resolve the difficulty of direction finding in the strong impulse noise, the CS-IML approach was proposed. The proposed CS-IML is a robust and effective approach for direction finding in the strong impulse noise. Monte-Carlo simulations have shown that CS-IML is superior to previous classical approaches of direction finding and has the capability to find independent and coherent signal sources in the strong impulse noise. Moreover, the CS-IML is a promising approach for application, and our future work will focus on improving the performance of CS-IML by combining other meta-heuristics in the strong impulse noise.

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